Analysis of Visualisation Requirements for Fuzzy Systems

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Abstract

This paper provides a comprehensive analysis of the working and requirements of fuzzy systems with the view to devise appropriate visualisation framework and techniques for these systems using a user- and task-oriented approach. We firstly discuss the nature of fuzzy data and the essential components of typical fuzzy systems, then categorise different visualisation requirements from three perspectives: user of fuzzy systems, designer of fuzzy systems and designer of visualisation systems. The visualisation framework also include mechanisms for capturing users' profiles in order to customise the system to their own needs. We then examine how different visualisation techniques can be adapted to satisfy these requirements. Motivations for an architecture of a visualisation system which is based on a multi-agent approach are also presented.

CR Categories: I.3 Computer Graphics; I.5.1 Fuzzy Sets.

Keywords: visualisation techniques, fuzzy data, fuzzy rules, fuzzy systems.

1 Introduction

Visualisation has been recognised as a viable methodology to enable users to interpret large amounts of data and to gain deeper insights into the nature and working of complex systems. Much research work has been carried out over the last decade to develop techniques, tools, and software architecture to facilitate understanding and decision-making process. The bulk of such work so far has been focused on those systems which involve crisp data and crisp relationships. However, many real world applications involve fuzzy data, fuzzy variables and fuzzy relationships. For example, to analyse the sustainability of a natural environment for planning purposes would involve the understanding of the characteristics of all species (animals, plants, etc.), their activities and the impacts of these activities upon the environment. Data collected on species and their activities is only accurate within a certain degree of precision. Furthermore, information derived from expert knowledge is often qualitative in nature. It is often difficult for users to understand the structure and characteristics of data, trends and impacts due to changes to the systems and interactions between different components of the systems. The fuzziness or uncertainty in data and relationships brings another dimension of complexity to the visualisation problem. We not only wish to highlight the patterns, trends and interactions inherent in data and relationships, but also wish to find effective ways to interpret the implication of their impreciseness, the impacts of the propagation of such impreciseness and the level of confidence in the outcomes obtained at every stage. Since the quality of the outcomes of a fuzzy system depends on the ability to retain the fuzziness at intermediate stages to prevent loss of useful information, the last two tasks are of particular importance.

Research in visualisation of fuzzy systems is still at an early stage. A few current approaches have some limitations due to either their ad hoc nature, or their ability to deal with only a specific aspect of the problem of visualisation of fuzzy systems (e.g. Behold & Holve 2000, Cox et al. 2001, Dickerson et al. 2001, Jiang 1998, Hall & Berhold 2000, Nurnberger et al. 1999 and 2001). In addition, visualisation methods are often focused on data sets and only loosely coupled with the analytical process. It is left to users to decide how they deploy those visualisation tools provided. For an inexperienced user, this might mean many trial-and-error attempts to determine how best to obtain insight into specific tasks. The usefulness of a visualisation system would therefore be enhanced if it is driven primarily by those tasks that need to be performed, and not by data sets because such a system would link more tightly with the analytical process which underpins human understanding and decision making. Another aspect needs to be considered is how to cater for different types of users. The needs of users of a fuzzy system are very different from those who design such a system, or from those who design the visualisation system.

The aim of this paper is to examine the problem of visualising fuzzy systems in a more holistic manner, with the view to develop a systematic framework based on a higher level of abstraction, where the visualisation is driven by users' needs in terms of application tasks and personal view points. Within such a framework, data would be organised according to task requirements. Search and navigation methods and tools would be more context-sensitive and would operate only on relevant information subspace. Rules on how information can be integrated from different sources would be well-defined and linked closely with events. It would also be beneficial to provide a mechanism for relevance feedback to capture users' views and refine the visualisation to suit. Such information can be used to construct users' profiles in order to customise for their needs.

Section 2 firstly discusses the sources of fuzzy data and their causes, then analyses the characteristics of typical fuzzy systems in terms of their components and tasks performed. Section 3 examines the visualisation requirements from different users' and tasks' perspectives. In Section 4, visualisation techniques are categorised and examined for their suitability for extending to fuzzy systems. In Section 5, we discuss the reasons that motivate the development of an agent-based framework for visualisation and how such a framework can be organised and implemented.

2 Characteristics of Fuzzy Systems

Commonly available information can be classified into three groups: *factual* information which is numerical and measurement-based; *pseudo-measurement* and pseudo-numerical information (e.g. "this model is available in the 60's"); and *perceptual-based* information which is mainly linguistic, but is also available in other forms such as image and sound-based (e.g. "this engine is nearly the end of its useful life") (Zadeh 1997). Uncertainty which may occur in all these information groups come from many sources. Table 1 shows typical sources of information uncertainty and their causes.

Sources of information uncertainty	***Causes***
Limited accuracy	Limitation in measuring instruments, or computational processes, or standards.
Missing data	Physical limitation of experiments; limited sample size or non-representative sample.
Incomplete definition	Impossibility or difficulty in articulating exact functional relationships or rules.
Imperfect realisation of a definition	Physical or conceptual limitation.
Inadequate knowledge about the effects of the change in environment	Model does not cover all influence factors; or was made under slightly different conditions; or was based on views of different experts.
Personal bias	Differences in individual perception
Ambiguity in linguistic descriptions	A word may have many meanings; or a state may be described by many words.
Approximation or assumptions embedded in model design methods or procedures	Requirements or limitations of models or methods.

Table 1. Sources and causes of information uncertainty.

Since the sources and causes of uncertainty are different, various models are required to faithfully represent different types of information. In a previous paper, we discussed the suitability of three types of model for this purpose: statistical, fuzzy and probability models (Reznik & Pham). Within the context of this paper, we assume that all three types of model can be used.

In order to design an effective generic framework for visualisation of fuzzy systems, we need to understand their essence: what they are composed of, how things are related to each other, and what activities are being performed. We now categorise the components of a typical fuzzy system.

Entities

There are two main types: physical entities (e.g. animals, fauna); abstract entities (e.g. sustainability). However, as users can interact with the system and influence the way the system works, they may also be considered as entities of the system.

Data Objects

Data objects may have different types of representations: numerical, symbolic (e.g. rules), visual (e.g. diagrams, graphical objects, images), audio.

Relationships

One of the most important tasks of a visualisation system is to facilitate the understanding of relationships that underpin the working of a fuzzy system. We categorise these relationships into 5 main types:

- Data-data (e.g. data fusion, integration, transformation);
- Data-task (e.g. different views of input data for different tasks; different tasks produce different types of output data);
- Data-user (e.g. different users may have different views or preferred ways to manipulate data and extract information);
- Task-task (e.g. the way a task is performed influences how a subsequent task is performed);
- User-user (e.g. users may share, compare, modify, or correct knowledge, or negotiate based on information each of them possesses).

Events

An event changes the state of the system, hence it is important to note and record events that significantly influence the performance of the system. We categorise events into 3 main types:

- Pre-scheduled according to an independent factor (e.g. time);
- As a result of user's interaction;
- Automatically spawned from another event according to some assumptions or constraints.

Tasks

Since our aim is to design a user- and task-oriented visualisation framework, it is essential to clearly identify the types of task in order to find suitable visualisation techniques as well as to design the flow of visualisation tasks. To distinguish their degree of complexity, we categorise tasks into 2 types: low-level and high-level.

- Low-level tasks: computing numerical data, degree of fuzziness, rules (aggregation, implication, de-fuzzification, belief, evidence, Bayesian probabilistic calculus). The results of the low-level tasks may be used as input to high-level tasks.
- High-level tasks: finding unusual patterns, trends, triggers of important events, dependency in relationships (data mining); correcting unwanted behaviour; providing feedback; learning from mistakes (eg. by creating new rules); optimising system given some constraints (eg. selecting good level of fuzziness for each variable); forming a predictive model based on past experience.

Outcomes

Information of interest on the final outcomes of a fuzzy system includes the level of acceptance of quality, degree of confidence, and degree of impreciseness of the outcomes.

3 Requirements for Visualisation of Fuzzy Systems

We examine visualisation requirements for fuzzy systems from the user- and task-oriented point of view, where a user wishes to be able to interact and select on the fly what to visualize and how to do it according to the results of current task being perceived from their own point of view. In other words, visualisation methods are neither fixed in advance nor operated on precomputed data. Instead, visualisation is interwoven with the tasks being performed in a fuzzy system so that the user can gain more insight and improve the decision-making process. Thus, there should be options for users to request extra tasks to be performed in order to generate data as required. We now discuss in more detail the requirements of different types of users and the goals of visualization based on these requirements.

Users of visualisation systems

We categorise the users of visualization into three main types:

- *Users of fuzzy systems*: usually wish to be able to interpret the data, to know its special features and the reliability of results. They also wish to be able to have more confidence in each decision and to understand the implication of each intermediate decision. This understanding would facilitate the finetuning of each result. Another capability these users would appreciate is to set up 'what-if' scenario in order to have insights into the impacts and to predict outcomes given certain constraints. At a more advanced level, they may wish the system to be able to capture their individual needs and preferences and to modify its services to suit them.

- *Designers of fuzzy systems*: require information on the internal structures of these systems for planning, verification and analysis. These include the structures of rules, clustering effects, contributions of rules during operation and the effects of different operators and rules on each task. These designers also wish to seek for conditions under which an optimal outcome is obtained at each stage or at the final stage.

- *Designers of visualisation systems*: usually wish to be able to evaluate the effectiveness of visualization techniques, to obtain feedbacks from users, to find drawbacks and to continuously improve the systems. These designers also wish to understand how the users of the visualisation systems (both as the users and as the designers of fuzzy systems) make use of visualisation with the aim to provide more suitable techniques and tools.

Major Visualisation Tasks

From the analysis of the requirements of different types of users, it is recognized that there exists some commonality that could be exploited in order to design an effective generic visualization framework. We categorise visualization tasks into four main types.

For interactive exploration

For each fuzzy variable or fuzzy rule, showing

- its degree of uncertainty
- effects on the task by varying its value
- effects on the task by varying its degree of uncertainty

For two (or more) fuzzy variables or fuzzy rules, showing

- their inter-dependency (or relationships) including extreme or salient behaviour
- their inter-dependency regarding the degree of uncertainty
- effects on the task by varying the value and degree of uncertainty of each variable or rule
- effects on the task due to changes in the ways fuzzy rules are performed (e.g. different operations for aggregation, implication, de-fuzzification)

For automatic computer-supported exploration

- automatic notification of special patterns, salient characteristics given specified conditions (or definitions).
- highlighting unusual results.
- visualising the quality and the degree of uncertainty of each outcome (e.g. numerical results, decisions).
- displaying proposed alternatives.
- comparing current results with previous ones.
- providing optimisation for certain tasks under specified constraints and visualision of this process.
- providing a mechanism for users to write scripts to perform a series of exploration tasks (e.g. via a visual language).
- providing common statistical analysis in visual forms.

Receiving feedback from users

- receiving instructions on tasks to be performed at the start of visualization and during intermediates stages.
- receiving input parameters, variables, constraints.
- receiving users' preferences, subjective judgements and desired degree of fulfillment of outcomes in qualitative forms.

Capturing users' profiles and adaptation

- recording patterns of tasks and subtasks performed by a user; of patterns of data and rule usage; choice of visualisation methods.
- re-prioritising tasks and data organisation to suit.
- automatically providing tasks and subtasks according to detected patterns.

4 Visualisation Techniques for Fuzzy Data and Fuzzy Rules

Successful visualisation of data is facilitated by the correct choice of visual features used to illustrate the magnitudes of data dimensions. The visual features are often chosen based upon their ability to act as a visual metaphor for the underlying data being represented (Keller & Keller 1993). With this in mind, we have examined two major components of visualisation: the visual features used and their organisation into higher representations, with the aim to extract appropriate visual representations for the visualisation of fuzzy data. We note that a n-dimensional fuzzy rule may be considered as a fuzzy point by cutting through a n-dimensional space.

4.1 Relevant Visual Features

Common visualisation techniques map various visual feature dimensions to data variables in order to highlight differences, to make comparisons, to show temporal effects, etc. We now delineate these features in turn and then show how they can be mapped to the level of imprecision within the data and be thus applied to the representation of fuzzy data. The features to be considered are: colour, luminance, size, transparency, depth, texture, glyphs, particles and blur.

Hue

Hue is heavily used to highlight data that is different, or to represent gradients in the data (Keller & Keller 1993, Tufte 1983). It can be used in a number of ways to represent fuzzy scalar data:

- Saturation of the hue can be used to highlight the precision or certainty of the data. The more saturated the hue, the more certain or crisp the value contained in that region is (Jiang 1998).
- Pastel, or low saturation regions, have the appearance of washing into each other and can be used to indicate the fuzziness of spatial region boundaries (Jiang 1998).
- The number of hue groups used in the mapping of values (*cardinality*) can indicate the level of precision in the values. This is analogous to the flat and Gouraud shading carried out in 3D graphics. The less precise solution has fewer variations in hue values, while a more precise solution has a smoother shaded appearance.
- Bad hue choices can be used to indicate the location of uncertainty via a lack of background/foreground separation, eg. red on purple. In most cases, this approach should be avoided, however, the lack of background/foreground separation can be a useful metaphor for uncertainty as the region may only just verge on being distinct, due to the proximity of the hue of the region to its background hue (Wandell 1995).

Luminance

In a similar manner to hue, luminance may be used to signify categories and highlight differences within scalar data.

- Foreground and background effects could be used to show the appearance of entities within the data, ie. the data could hover around JND (Just Noticeable Difference) values to indicate the ambiguity (Wandell 1995).
- The cardinality of the luminance feature could be varied to show the precision of the data in a similar manner to the cardinality of the hue space.

Size

Glyphs involving the size of the objects are often used to indicate the scalar component of vector information. An example of this is the variation of the size of error bars within to indicate the imprecision of the data point (Tufte 1983).

Transparency

This is similar to the concepts of blending a colour with a background, except that the background can be any object behind the present object being rendered. Transparency can be used to show underlying structure, but in this context can be used to show the fuzziness of the data by mapping the possibility of the fuzzy variable to the transparency.

Depth

Depth can be used to indicate an order or spatial positioning for the data.

- Data which is presented in stereo may have the algorithm modified to change the binocular fusion of the object to indicate fuzziness with the depth position of the data.
- Depth of field effects can be used, in a related manner to spatial blur caused by removal of high frequency information.

Texture

Texture may be applied to objects to indicate the level of precision, ambiguity or fuzziness in the spatial location upon an object or upon a spatial location. However, ambiguous texturing effects are usually given the title 'chart junk' (Tufte 1983) and are normally to be avoided due to problems with visual clutter.

- Differences in colour and luminance and shape textures could be used to indicate the presence of ambiguous data.
- Certain shimmering effects, usually to be avoided in visualisation, but could be used to indicate the presence of ambiguity within the region (Thomas 1997).

Glyphs/Icons

Both glyphs and icons can create a problem and a possibility, as they allow the representation of data using an object or shape etc. This leaves an unending list of possible glyphs to use with regards to visualisation of fuzzy information. Words could also be used in this application, due to fuzzy terms being the currency of such rule-based systems. Words, along with other complex icon-like glyphs, have been used in visualisation applications.

Particles

Particles could be used to represent the fuzziness of a region or an object by varying the space between them, and the colour of the particles themselves. The particles could also be rendered with motion blur to again indicate the level of data imprecision. Cartography often uses a form of this by drawing dashed lines to represent imprecise lines and boundaries, or by using different dot densities to represent shading effects (Goodchild et al. 1998).

Blur

Blurring or depth of field effects from spatial frequency components being removed in the image plane can be used to show the indistinct nature of data points (Gershon 1992, Kosara et al. 2002).

4.2 Higher Spatial Representations of Data

The visual features listed previously are usually spatially arranged to form a coherent display in graphic forms which enable the perception of various patterns in the data. We categorise these techniques into 7 main types:

2D Representations

2D graphs of various forms can be used to encode the colours and shapes into a display on a Cartesian system, in order to show the spatial relationships of values. These graphs may not necessarily related to a spatial locations. Some examples of graphs are: histograms, bar charts, tree diagrams, time histories of 1D slices, maps, iconic and glyph-based diagrams. The structure and inter-relationships of rules may be illustrated by graphs, trees and flowcharts.

Variation in intensity or colour may be used to encode another dimension on a 2D graph which indicate the degree of impreciseness or fuzzy membership functions of the data displayed. Graphs may also be used to represent the fuzzy membership functions or alpha-cuts of a fuzzy set.

Another common technique is to project data for reduction of dimensionality (e.g. Principle Component Analysis) and display results on a scatter plot. However, although this technique provides a high level analysis of the most significant components of the data, it has a drawback due to the loss of information during the process.

Other techniques such as multi dimensional scaling (Berthold & Holve 2000) and parallel coordinates (Hall & Berthold 2000) provide ways to display multi-dimensional fuzzy data in 2D without losing any information. For multi-dimensional scaling, the authors introduced an algorithm to generate 2D view of a set of fuzzy rules which minimizes the inter-point distances. The rule set is then visualized as a 2D scatter plot, where different grey scales denote different classes and the size of each square denoting each class indicates the number of examples and hence the importance of the class. For the parallel coordinates approach, n Cartesian coordinates are mapped into n parallel coordinates and a n-dimensional point becomes a series of (n-1) lines connecting the values on n parallel axes.

At the end of this Section, as an example, we shall illustrate how the visualisation of fuzzy rules by parallel coordinates provided by these authors could be improved to make it more intuitive for users by judicious choice of visual features.

3D Representation
A 3D volume has spatial regions mapped to a location in n-dimensional space. The features of the volume partitions could be modified to indicate the precision of the data within the volume (e.g. varying intensity, colour saturation, texture, opacity). These techniques may be used to show classification boundaries in fuzzy classification methods. 3D height-field (may be expressed as surfaces) could also be used to represent fuzzy membership functions of data displayed in 2D graphs.

To visualize hierarchical information, a cone tree method was introduced to represent a tree structure (Robertson & Mackinlay 1991). This techniques is later used to produce 3D flowchart to represent rule structure in a rule-based program to facilitate its understanding (Fujiwara et al. 1998). To extend these techniques to fuzzy rules, visual features described in the previous subsection can be integrated to the cone tree structure to express the degree of uncertainty in each rule (e.g. each node is displayed with different degree of opacity).

Parametric representations
Different parameters could be used to highlight or suppress various factors in an interactive manner. This method may also be performed in a non-interactive manner as a movie, using fixed temporal effects. This is useful from a computer human interfaces perspective as the imprecision in the data could be visualised over a number of perceptual feature dimensions to reinforce various combinations, and to allow interaction as another form of visualisation technique.

Dynamic representations
Various visual features discussed in the previous subsection could be used to modify the animation to display object behaviour over time, e.g. using motion blur levels, flickering etc. to represent the precision of the measurements of the object motion in a plane crash simulation.

Metaphors
As human can perceive the effects of certain common phenomena at a very fast speed, abstract representations may be used as metaphors to represent data that is not easily visualised. For example, expressions on human faces can be used to represent the quality of the results, where a happy / sad expression indicates good / bad quality.

Multimedia sensors
Haptic and audio feedback can be used to indicate precision, imprecision, eg. mapping mouse location to a form of sound that is noisy and incoherent in imprecise regions, and coherent and tonal in regions that are precise.

An Example
We now illustrate how the techniques described in this Section could be deployed judiciously in order to provide an intuitive and unambiguous visualization. To this end, we use the example of visualization of fuzzy rules applied to the Iris data provided by Hall & Berthold (2000). We discuss the techniques used by the authors and suggest other alternatives that would provide better perception of the results.

The authors generated 11 fuzzy rules and 3 classes from a training set of 75 plants in the Iris database, using 4 features for each plant. The centres of the cores of these rules are displayed in parallel coordinates, where fuzziness of points is indicated by the thickness of line or grey level. To distinguish different classes, the rules of the same class are displayed in the same grey levels (Figure 1). These techniques have some drawbacks due to the difficulty of visually distinguishing fine grades of grey level, especially on single lines. Furthermore, it is not possible to perceive the core and support of a fuzzy set simultaneously.

A better alternative is to extend to a 3D representation, where the fuzzy coordinates are displayed on the x-y plane and the z coordinate is deployed to indicate the fuzzy membership functions. These membership functions could be displayed as line segments (Figure 2) or contours (Figure 3). The separation of classes based on the confidence of the decision may be highlighted by distinct colours or filled polygons (Figure 2). Different alpha-cuts of the fuzzy rules may also be easily isolated by applying horizontal cutting planes through the 3D volume, and may be represented as translucent planes in the visualisations.

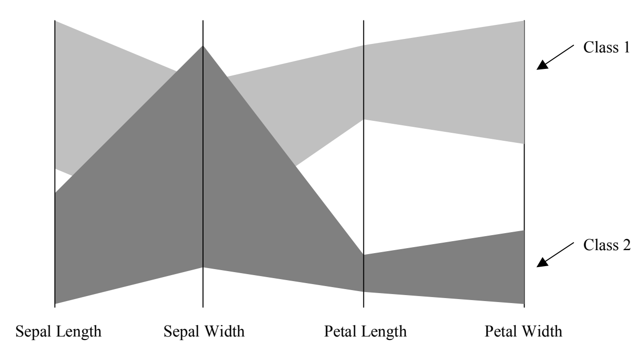

Figure 1 Illustration of 2D method developed by Hall and Berthold, for representing multidimensional fuzzy rules using fuzzy parallel coordinates (Hall & Berthold 2000). Two rules are illustrated from their Iris data example.

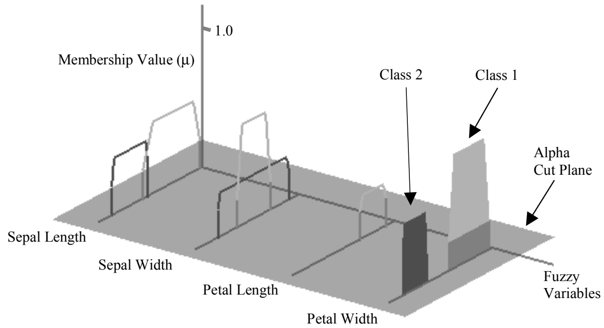

Figure 2 Illustration of the new 3D parallel visualisation showing the membership functions from Figure 1, with a superimposed alpha cut plane. The filled polygons highlight membership functions which classify with a high degree of confidence.

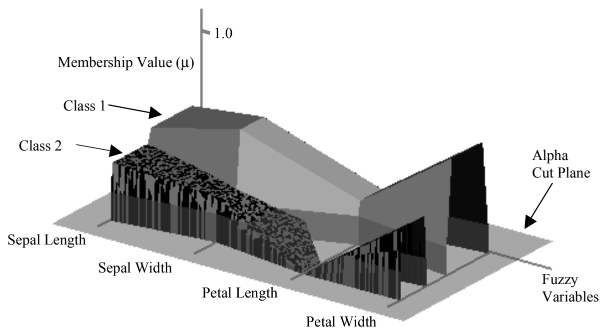

Figure 3 Visualisation of the Iris data with lit and textured surfaces showing the same Iris data. Note how the alpha cutting of the membership function for Rule 2 on the Petal Length dimension is easily perceived.

5 An Agent-based Visualisation Framework

Multi-agent approach has been increasingly adopted for application domains because it provides an effective way to coordinate activities and their interactions in a complex system to satisfy some common goals. An agent in our context is a computer program that can gather data about the environment, interpret the data and modify its behaviour to reflect the requirements of the environment. These capabilities are essential to satisfy the requirements of our visualization system. We proposed a visualisation framework based on 5 classes of agents: control agent, computation agent, symbolic agent, visualisation agent and profile agent. Figure 4 shows a schematic diagram of the architecture of system.

The control agent receives users' input which include specifications, queries and parameters. Based on such input, this agent distributes tasks to appropriate agents. It also receives results and demands from other agents when a task is completed or when further information is needed. Another duty for this agent is to generate new tasks if required based on the results sent by other agents. The control agent may be viewed as a representative of the user in an automatic mode. In our model, the user can be included in the loop and allowed to intercept the control agent in order to give different instructions if desired. The user can also intercepts other agent to select different methods for performing an operation instead of the default ones built in the system . The computation agent performs all numerical computation required by the system (e.g. statistics, probabilistic calculus, rough set operations, fuzzy set operations). It receives instructions from both the control agent and the visualisation agent. The symbolic agent makes use of the knowledge base to performs rule inferencing. It receives instructions from both the control agent and the visualisation agent. The visualisation receives instructions from the control agent and request information from the computation agent and rule agent in order to select appropriate visualisation techniques to provide displays. The results of the display then trigger the control agent or the user to issue another task. Another cycle then continues.

The profile agent records the pattern of the user's behaviour in terms of the selection of tasks, visualisation techniques, numerical methods or inference rules. Based on this information, the profile agent then modify the instructions issued by the control agent (e.g. re-prioritise tasks, change preferences, modes of display, etc.).

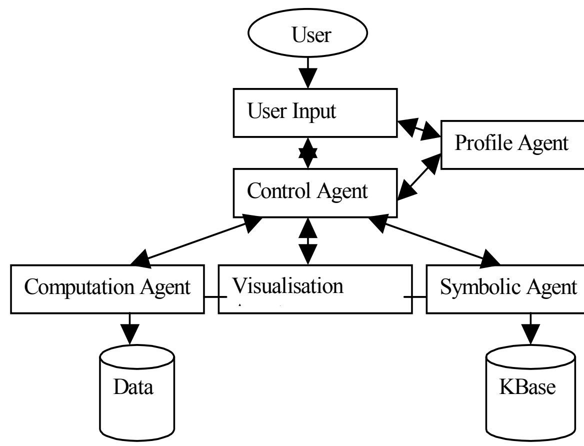

Figure 4 . Agent-based Visualisation System

3 Conclusions and Future Work

We have presented a comprehensive approach for constructing a visualization framework and techniques for fuzzy systems. This approach is based on the design of fundamental ontologies which underpin the structure and requirements of these systems. Our intention is to drive the visualisation from the perspectives of the users and tasks, rather than by the data itself. This framework will be implemented as a multi-agent system which facilitates the organization and flow of complex tasks and their inter-relationships and their interactions with the users. On-going work includes the articulation of the structure and activities of each of these agent classes and their implementation.

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